

PRESSURE COEFFICIENTS OF ELASTIC CONSTANTS FOR POROUS MATERIALS: CORRECTION FOR POROSITY AND DISCUSSION ON LITERATURE DATA

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An empirical scheme, by which one can correct porosity effects on pressure coefficients of elastic constants for porous materials, is presented. The underlying idea for the scheme is our recognition of experimental observations that the rate of change of ultrasonic transit-times with hydrostatic pressure in a porous medium is independent of small porosity at a pressure range of 2 to 10 kb, a range of pressure most commonly utilized in acoustic experiments. The scheme has been successfully tested with two polycrystalline forsterite samples of 1.65% and 6.09% porosities, two polycrystalline corundum samples with about 0.3% porosity each, and one polycrystalline rutile sample with 0.89% porosity, for all of which the corresponding single-crystal data are available. In the light of the present work, most of the elasticity data obtained on polycrystalline samples may require revision; for example, the literature value of dK_s/dp for hematite with 0.4% porosity is 4.53, which after the porosity correction becomes 4.91, while dK_s/dp reported as 4.80 for a forsterite sample with 6.09% porosity becomes 5.19 with the porosity correction. It is concluded that the scheme presented here may be useful as a working tool for experimentalists dealing with the elasticity of polycrystalline materials.

1. Introduction

One of the more important objectives of geophysics is to transform, uniquely, the elasticity of the Earth into parameters of composition, pressure, and temperature of earth materials. Accurate data on these equation-of-state parameters measured in the laboratory on carefully characterized minerals and rocks are therefore essential in the interpretation of geophysical field-data as well as in our understanding of solid state properties of these materials. Laboratory measurements of these data can be made either on single-crystals or on dense-formed polycrystalline samples. The frequent limitation of the unavailability of single-crystal specimens large enough for acoustic measurements to be made upon them hinders the laboratory determination of these data.

With advances in ceramic fabrication process (see, for example, Crandall et al. [1]), some polycrystalline aggregates can be fabricated into quasi-isotropic bodies with a density comparable to the single-crystal

density*. Clearly, however, most polycrystalline

* Since the writer's success in 1961 in hot-pressing corundum samples with small porosities and the subsequent correlation for a correspondence between single-crystal and polycrystalline elastic properties (Chung [2] and see also Anderson [3, p. 912; 4, p. 491]) much progress has been made regarding the elasticity of polycrystalline oxides and silicates. O.L. Anderson and his associates at Lamont-Doherty Geological Observatory made use of this idea extensively (see, for example, ref. [19] for a summary). The following examples in chronological order of development may be listed: (a) *periclase* with about 0.1% porosity, see Chung [5, 33], Anderson [4, p. 51], and Anderson and Schreiber [6, 34]; (b) *spinel*s with about 1% porosity, see Chung et al. [7]; (c) *forsterite* with 6.09%, see Schreiber and Anderson [8]; (d) *zincite* with about 1% porosity, see Chung and Buessem [9], and also Soga and Anderson [10]; (e) *lime* with 1.8% porosity, see Soga [11]; (f) *hematite* with 0.4% porosity, see Liebermann and Schreiber [12]; (g) *rutile* with 0.89% porosity, see Chung and Simmons [13]; (h) *quartz* with 0.15% porosity, see Chung and Simmons [13]; (i) *olivine* of various (Fe/Mg) ratios, see Chung [14]; (j) *fayalite* with about 2.5% porosity, see Fujisawa [15] and Chung [14]; (k) (Fe₂SiO₄) *spinel*, see Mizutani et al. [16].

samples prepared by hot-pressing or sintering method contain residual pores along the interstices of adjacent mineral grains; the porosity may frequently be as high as 5 to 10%, although values of a few percent or less are more common. When acoustic measurements are made on porous polycrystalline samples, the values must be corrected before being used to predict the intrinsic behavior of material deep in the Earth, which undoubtedly is pore-free.

Effects of porosity upon pressure derivatives of the elastic moduli are complicated not only by the manner in which the porosity-sensitive moduli change with pressure, but also by the change of porosity with pressure. In an earlier discussion of this problem, Chung and Simmons [17, p. 5318] suggested that for porosity less than 1% ($d \ln M / dp$) = ($d \ln M^0 / dp$) is a good approximation (where M is a modulus and the superscript $(^0)$ refers to the value at zero-porosity. Based on Mackenzie's work [18], Anderson et al. [19, p. 507] found (by differentiating Mackenzie's expression for the bulk modulus) an expression for dK_S^0 / dp in terms of dK_S / dp measured on a porous sample. Walsh [20] in his recent analysis on the same problem pointed out that Mackenzie [18] developed his theory using linear elasticity in which K_S^0 and μ^0 are independent of stress and that a simple differentiation of Mackenzie's expression as was done in [19] could not result in a correct expression for dK_S^0 / dp . Walsh [20] then used Murnaghan's theory of finite strains to find expressions for dK_S^0 / dp in terms of the second and third-order elastic constants of the solid material. Use of Walsh's expressions for porous materials requires these high-order elastic constants of solid under discussion; frequent unavailability of these constants limits further application.

In this paper, the writer suggests a practical method for correcting porosity effects on pressure coefficients of the elastic parameters of porous materials. The present scheme has been successfully tested with two independent sets of porous forsterite samples having different porosities. The scheme was also tested with porous corundum and rutile samples successfully. The scheme described here may be a useful tool for experimentalists working with polycrystalline materials.

2. Scheme

The quantity of interest in the acoustic experiments, in which pressure is a variable, is the first derivative of an isotropic elastic modulus M with respect to hydrostatic pressure evaluated at zero-pressure; this will be denoted here after as $\{dM/dp\}_{p=0}$. This derivative is an isothermal one, although the velocity-of-sound measurements involve an adiabatic process. Thus, the acoustic data resulting from such experiments are *thermodynamically mixed* isothermal pressure derivatives of the adiabatic modulus. For a modulus M_j , where the subscript j refers to either compressional or shear mode, we have

$$M_j = 4\rho \ell^2 (f_j)^2 \quad (1)$$

where ρ and ℓ are the density and length of the specimen at the initial condition, respectively, and f_j is the corrected pulse repetition frequency in the j th mode. Taking logarithms and differentiating both sides with respect to pressure,

$$\frac{d \ln M_j}{dp} = \frac{d \ln \rho}{dp} + 2 \left(\frac{d \ln \ell}{dp} \right) + \frac{d \ln (f_j)^2}{dp} \quad (2)$$

Since $(d \ln \ell / dp) = (d \ln V / dp) / 3$ for the isotropic medium and $(d \ln \rho / dp) = -(d \ln V / dp) \equiv 1/K_T$, where V is the volume, and after evaluating the derivatives at zero-pressure, we have

$$\left\{ \frac{dM_j}{dp} \right\}_{p=0} = \left\{ \frac{M_j}{3K_T} \right\}_{p=0} + \{M_j R_j\}_{p=0} \quad (3)$$

where $R_j = d(f_{jp}/f_{j0})^2 / dp$ and this is obtained by fitting $(f_{jp}/f_{j0})^2$ versus pressure data to a straight line by the method of least squares. K_T is the isothermal bulk modulus and it is related to the adiabatic bulk modulus K_S by $K_T = K_S / (1 + \alpha T \gamma_G)$, where α is the coefficient of volume expansion, γ_G is Grüneisen's ratio, and T is temperature in $^{\circ}\text{K}$. Thus it is clear from eq. (3) that the measurements of isotropic compressional and shear velocities of sound at a reference temperature and ultrasonic pulse-repetition-frequencies corresponding to these

velocities as a function of pressure (also at the reference temperature) yield the values of $\{dM_j/dp\}_{p=0}$.

The use of eq. (3) for porous materials involves a two-step correction:

(a) For a polycrystalline sample with a known porosity, V_p and V_S are measured in the usual way. The elastic properties as one measures on a polycrystalline sample, are the *apparent* properties of the sample; they may or may not be corresponding to the *intrinsic* elastic properties of the sample being studied. Certain hot-pressed samples often contain microcracks, for example, and effects of these microcracks on the elastic properties of the samples should receive the careful attention of the investigator. Helpful references to these effects are Brace [21], Walsh [22], and Nur and Simmons [23]. The most commonly practised method of finding the intrinsic elastic properties of such samples is to measure both P and S velocities as a function of hydrostatic pressure to about 7 to 10 kb, as is frequently done (see, for example [13; 16, p. 2743]); from these $V_j(p)$ data, velocities at zero-pressure are found by extrapolation of high-pressure results back to the zero-pressure point. These velocities found at the origin in this manner correspond to the zero-pressure values for that porous sample; from these data, isotropic elastic properties at zero-porosity can be evaluated. Weil [24, p. 217] and Walsh [22] discussed how elastic properties of non-porous polycrystalline materials can be evaluated from the elastic data obtained on a porous sample. The Weil-Hashin expressions for the shear and bulk moduli with constants k_1 and k_2 , which are a function only of Poisson's ratio σ_S , are recommended. Mackenzie's [18] expressions can be used as well. Thus, for $\sigma_S = \sigma_S^0$, we obtain μ^0 , K_S^0 , V_p^0 , and V_S^0 .

(b) It was observed [17, 26] that the quantity R_j in the second term of eq. (3) is independent to the first order of small porosity at a pressure range of 2 to 10 kb, a range most commonly utilized in acoustic experiments. The theoretical justification for this observation is difficult, if not impossible, without making assumptions as to size, shape, and orientations of pores in the polycrystalline aggregate. In addition, even for a pore-free aggregate, the task of determining the microscopic state of stress distribution is hopelessly difficult, due to numerous superimposed effects which originate from the properties of the constituting mineral grains and from the boundaries between them. For these reasons, no satisfactory general model for the elasticity of po-

rous materials has yet been developed, in spite of numerous investigations (see for a review [24, p. 217; 22]). An earlier analysis [17, p. 5320], based on Walsh's work for the rate of change of pores with pressure, indicates that, for a polycrystalline corundum with a porosity $\theta = 0.3\%$, the quantity $(d\theta/dp)$ estimated at the origin is in the order of -3×10^{-6} per kb. This value is small, and it is well within the scatter of most experimental data; thus, suggesting the observation may be justified for small properties (say θ less than one or two percent).

Additional support for the observation made in the earlier paragraph follows from the work of Walsh [22] and Brace [21]. As shown by Walsh and Brace, the pressure p^* required to close a cavity having the aspect ratio a is

$$p^* = aY \quad (4)$$

where Y is Young's modulus of the solid material surrounding the cavity. The values of Young's modulus for oxides and silicates of interest to geophysics are at least 1000 kb or greater (see Birch [25]). If pores are spherical, as in many polycrystalline samples prepared by sintering or by hot-pressing, the aspect ratio a is one; this means then that the pressure required to close the pores is 1000 kb or higher, depending upon the stiffness of the materials. This is a very high pressure as compared to the range of pressure involved in acoustic experiments. It would appear then that an application of 2 to 10 kb pressure to a sample containing spherical pores is too small a pressure for pore closure to affect the quantity R_j .

Thus, with experimental quantity R_j determined on a porous sample, one should be able to find the pressure coefficients of compressional, shear, and bulk moduli of the non-porous material from eqs. (5) and (6).

$$\begin{aligned} \left\{ \frac{dM_j^0}{dp} \right\}_{p=0} &= \left\{ \frac{M_j^0}{3K_T^0} \right\} \\ &+ \left\{ M_j^0 \cdot \frac{d}{dp} (f_{jp}/f_{j0})^2 \right\}_{p=0} \end{aligned} \quad (5)$$

Table 1
Isotropic elastic properties of forsterite.

Sample property	Unit	Polycrystalline data*				Single-crystal data**	
		Porous	Nonporous	Porous	Nonporous		
ρ	g/cm ³	3.021	3.217	3.164	3.217	3.222	3.224
V_p	km/sec	7.586	<u>7.608</u>	8.459	<u>8.534</u>	8.594	8.569
V_S	km/sec	4.359	<u>4.369</u>	4.938	<u>4.977</u>	5.033	5.015
ϕ	(km/sec) ²	32.24	<u>32.42</u>	39.03	<u>39.82</u>	40.08	39.89
σ_S	none	0.254	<u>0.240</u>	0.241	<u>0.240</u>	0.242	0.240
L_S	kb	1739	<u>1862</u>	2264	<u>2343</u>	2379	2367
μ	kb	574	<u>614</u>	772	<u>797</u>	816	810.8
K_S	kb	973.6	<u>1043</u>	1235	<u>1281</u>	1291	1286
dL_S/dp	none	6.53	<u>6.94</u>	7.27	<u>7.51</u>	7.40	7.77
$d\mu/dp$	none	1.30	<u>1.32</u>	1.80	<u>1.85</u>	1.82	1.80
dK_S/dp	none	4.80	<u>5.19</u>	4.87	<u>5.04</u>	4.97	5.37
R_p	per kb		0.00341		0.00294	—	—
R_S	per kb		0.00182		0.00206	—	—
Reference †		A		B		C	D

* Data entered under "porous" are the original experimental values reported by authors cited, and values listed under "non-porous" are corrected for porosity in accordance with the scheme presented in this paper.

** VRH values based on single-crystal data reported by authors cited.

† (A) Schreiber and Anderson [8]. Based on the same data, Anderson et al. [19, p. 494] list the value of $(dK_S/dp) = 4.87$ while in another report by Soga and Anderson [10, p. 2987] it gives 5.08. To avoid the apparent confusion, we used the original data presented by Schreiber and Anderson [8, p. 763]. (B) Chung [26]. (C) Graham and Barsch [27]. (D) Kumazawa and Anderson [28].

$$\left\{ \frac{dK_S^0}{dp} \right\}_{p=0} = \left\{ \frac{dL_S^0}{dp} \right\}_{p=0} - \frac{4}{3} \left\{ \frac{d\mu^0}{dp} \right\}_{p=0} \quad (6)$$

where L_S^0 , μ^0 ; and K_S^0 are the porosity-corrected values from step (a) and the superscript (°) refers to the zero-porosity.

3. Testing of the scheme: examples.

3.1. Forsterite

Table 1 presents the isotropic elastic parameters of two porous samples of forsterite before and after the porosity correction. The first set of these acoustic data is due to Schreiber and Anderson [8] and the second set to Chung [26]. The polycrystalline sample

used by Schreiber and Anderson has 6.09% porosity, whereas Chung's sample contains 1.65% porosity [14]. To be noted here is that values of dK_S/dp for porous forsterite samples found in [8] and [26] are 4.80 and 4.87, respectively; with the porosity correction according to the present scheme, these values become 5.19 and 5.04. As they are clearly seen from table 1, the porosity-corrected values compare very well with 4.97 [27] and 5.37 [28], the corresponding values determined on gem-quality forsterite single-crystals. The elasticity data at ambient conditions reported originally by Schreiber and Anderson [8] differ from the single-crystal data reported by Graham and Barsch [27] and Kumazawa and Anderson [28] as well as from Chung's [26] polycrystalline data. The probable reason here may be due to the presence of a secondary phase in their "forsterite" sample. The

Table 2
Isotropic elastic properties of corundum.

Sample property	Unit	Polycrystalline data*				Single-crystal data**
		Porous	Nonporous	Porous	Nonporous	
ρ	g/cm ³	3.972	3.986	3.974	3.986	3.986
V_p	km/sec	10.845	<u>10.865</u>	10.845	<u>10.889</u>	10.847
V_s	km/sec	6.373	<u>6.383</u>	6.377	<u>6.398</u>	6.402
ϕ	(km/sec) ²	63.47	<u>63.72</u>	63.39	<u>64.00</u>	63.02
σ_s	none	0.236	<u>0.236</u>	0.234	<u>0.234</u>	0.233
L_s	kb	4672	<u>4705</u>	4674	<u>4726</u>	4690
μ	kb	1613	<u>1624</u>	1616	<u>1632</u>	1634
K_s	kb	2521	<u>2540</u>	2519	<u>2551</u>	2512
dL_s/dp	none	6.33	<u>6.36</u>	6.51	<u>6.57</u>	6.58
$d\mu/dp$	none	1.76	<u>1.77</u>	1.77	<u>1.79</u>	1.73
dK_s/dp	none	3.98	<u>4.01</u>	4.16	<u>4.19</u>	4.27
R_p	per kb		0.001222		0.001259	—
R_s	per kb		0.000958		0.000960	—
Reference †		A		B		

* Data entered under "porous" are the original experimental values reported by authors cited, and values listed under "non-porous" are corrected for porosity in accordance with the scheme presented in this paper.

** VRH values based on single-crystal data reported by Wachtman et al. [36] and Gieske and Barsch [30].

† (A) Schreiber and Anderson [29]. (B) Chung and Simmons [17].

experience of this author indicates that the presence of secondary phases affects considerably the second-order elastic constants, but effects of these secondary phases on the pressure derivatives (particularly the value of dK_s/dp) are relatively small [26]. The unusually low values of both P and S velocities and the elastic constants as observed by Schreiber and Anderson [8] on their "forsterite" sample are believed to be associated with the presence of "barium aluminum silicate", the presence of which was noted by these authors.

3.2. Corundum

Table 2 lists two independent sets of the isotropic elastic parameters of polycrystalline corundum samples before and after the porosity correction. The first set of these acoustic data is due to Schreiber and Anderson [20] on a Lucalox alumina sample with 0.35% porosity, and the second set is due to Chung and Simmons [17] on a hot-pressed alumina sample with

0.35% porosity (Chung [2]; Anderson [3, p. 912; 4, p. 491]). Two sets of the acoustic data are generally in good agreement. It is noted that, with the porosity correction in accordance with the present scheme, values of the pressure derivatives compare well. It is also noted that these porosity-corrected values of the pressure derivatives compare very well with the corresponding values found on single-crystal corundum [30].

3.3. Rutile

Chung and Simmons [13] reported acoustic data on two porous rutile samples, one with 25% porosity and the other with 0.89% porosity. The property measured under pressure on the sample with 25% porosity was considered anomalous*, and these data

* It was noted earlier that the measured elastic properties of sample 1 (with 25% porosity) were considered anomalous.

Table 3
Isotropic elastic properties of rutile.

Sample property	Unit	Polycrystalline data*		Single-crystal data**
		Porous	Nonporous	
ρ	g/cm ³	4.212	4.250	4.260
V_p	km/sec	9.146	<u>9.193</u>	9.261
V_S	km/sec	5.102	<u>5.122</u>	5.137
ϕ	(km/sec) ²	48.96	<u>49.53</u>	50.59
σ_S	none	0.264	<u>0.274</u>	0.272
L_S	kb	3524	<u>3592</u>	3654
μ	kb	1096	<u>1115</u>	1124
K_S	kb	2062	<u>2105</u>	2155
dL_S/dp	none	7.57	<u>7.7</u>	7.80
$d\mu/dp$	none	0.91	<u>0.9</u>	0.78
dK_S/dp	none	6.35	<u>6.4</u>	6.76
R_p	per kb	0.001984		—
R_S	per kb	0.000679		—

* Data from Chung and Simmons [13].

** VHR values based on single-crystal data reported by Manghani [31].

will not be used in the present discussion. Table 3 presents the acoustic data obtained on a rutile sample with 0.89% porosity. The porosity-correction has been made to these data, and they are compared in the table with the corresponding data of gem-quality rutile single-crystals [31]. The comparison between the porosity-corrected polycrystalline data and the single-crystal data is generally satisfactory.

4. Discussion and conclusion

Examples with forsterite, corundum, and rutile given above indicate that the scheme presented in this paper seems to correct porosity effects on the pressure derivatives of the elastic constants of porous materials.

One-fourth of the volume of this sample was pores; with application of pressure, the pressure medium (petroleum ether in this case) undoubtedly penetrates into the pores. The property measured in such a sample is then some property of a "solid-liquid" composite under the pressure. Examination of the original data (see [13], p. 136) reveals that the shear waves were more critically affected.

Validity of the proposed scheme is essentially established. The scheme could be a useful tool for experimentalists working with the elasticity of polycrystalline materials.

The elastic constants and their pressure derivatives of 10 compounds of interest to geophysical and geochemical theories have been tabulated by Anderson et al. [19]. Acoustic data on seven out of these ten compounds are based on porous polycrystalline samples they studied, in which the porosity ranges from 0.11% for periclase to 6.09% for forsterite. The writer applied the present scheme of correcting porosity effects to their acoustic data; the results are tabulated in table 4. Also entered in the table are the elasticity data by this writer for periclase, corundum, quartz, rutile, and forsterite. Effect of the porosity correction is generally small. Most severely affected quantities are $d\mu/dp$ and dK_S/dp for hematite (with 0.4% porosity) and forsterite (with 6.09%). The dK_S/dp value as reported originally by Liebermann and Schreiber [12] for hematite is 4.53, which after the porosity correction becomes 4.91, while dK_S/dp , reported originally as 4.80 by Schreiber and Anderson [8] for their for-

Table 4
Comparison of the elastic parameters before and after the porosity correction.

Material	Ref. *	Before correction						After correction				
		Density ρ (g/cm ³)	Porosity θ (%)	μ (kb)	K_S	$\frac{d\mu}{dp}$	$\frac{dK_S}{dp}$	ρ_0 (g/cm ³)	μ^0 (kb)	K_S^0	$\frac{d\mu^0}{dp}$	$\frac{dK_S^0}{dp}$
MgO	a, m	3.5797	0.11	1288	1624	2.62	4.58	3.583	1291	1627	2.74	4.43
MgO	b	3.582	0.03	1293	1644	2.41	4.28	3.583	1293	1644	2.41	4.28
CaO	c, m	3.285	0.30	761	1059	1.64	5.23	3.345	789	1099	1.69	5.42
ZnO	d, m	5.624	0.90	442	1394	-0.69	4.78	5.675	450	1437	-0.70	4.88
BeO	e, m	3.000	0.26	1618	2201	0.88	5.52	3.008	1626	2213	0.88	5.54
α -Al ₂ O ₃	f, m	3.972	0.35	1613	2521	1.76	3.98	3.986	1624	2540	1.77	4.01
α -Al ₂ O ₃	g	3.974	0.30	1616	2519	1.77	4.16	3.986	1632	2551	1.79	4.19
α -Fe ₂ O ₃	h, m	5.2539	0.40	910	2066	0.73	4.53	5.274	917	2088	0.89	4.91
α -SiO ₂	i	2.645	0.15	446	377	0.47	6.53	2.649	447	378	0.5	6.5
TiO ₂	j	4.212	0.89	1096	2062	0.91	6.35	4.250	1115	2105	0.9	6.4
Mg ₂ SiO ₄	k, m	3.021	6.09	574	974	1.30	4.80	3.217	614	1043	1.32	5.19
Mg ₂ SiO ₄	l	3.164	1.65	772	1235	1.80	4.87	3.217	797	1281	1.85	5.04
Fe ₂ SiO ₄	l	4.287	2.41	502	1144	0.64	5.49	4.393	536	1220	0.62	5.92

* (a) Schreiber and Anderson [34].

(b) Chung and Simmons [33].

(c) Soga [11].

(d) Soga and Anderson [10].

(e) Soga [35].

(f) Schreiber and Anderson [29].

(g) Chung and Simmons [17].

(h) Liebermann and Schreiber [12].

(i) Chung and Simmons [13].

(j) Chung and Simmons [13].

(k) Schreiber and Anderson [8].

(l) Chung [26].

(m) Anderson et al. [19, table 1].

sterite sample, becomes 5.19 with the porosity correction. Furthermore, as is evident from table 4, the $d\mu/dp$ value for hematite reported originally in [12] as 0.73 undergoes a 22% change with the porosity correction.

On the basis of their original experimental data obtained on ten model compounds, Anderson et al. [19, figs. 3 to 10] and Anderson and Liebermann [32] discussed patterns relating the elastic properties of oxides and silicates. Their extensive discussions were based on the *apparent* elastic properties of

various samples, including the seven porous polycrystalline specimens. *Any discussion of the general patterns relating the elastic properties requires the intrinsic properties of model solids characterizing the elasticity of those materials.* In view of table 4, then, all those correlations and patterns observed previously [19, 32] for the elastic properties of model oxides and silicates may have to be revised.

The conclusion of this paper is then that a more realistic establishment of the general patterns relating the elastic properties requires the *intrinsic* properties

of model solids characterizing the elasticity of those materials. In transforming the elasticity of the Earth into solid-state parameters of composition, pressure, and temperature, the properties needed are the intrinsic elastic properties of earth materials (not the apparent properties as one measures on a porous sample).

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Notation

<i>Symbols</i>	<i>Units</i>	<i>Meaning</i>
V_p	km/sec	Velocity of compressional waves
V_p^0	km/sec	Velocity of compressional waves at zero-porosity
V_s	km/sec	Velocity of shear waves
V_s^0	km/sec	Velocity of shear waves at zero-porosity
V_j	km/sec	Velocity of j th mode
M_j	kb	Elastic modulus of j th mode
M_j^0	kb	Elastic modulus of j th mode at zero-porosity
L_s	kb	Longitudinal modulus given by (density $\times V_p^2$)
L_s^0	kb	Longitudinal modulus at zero-porosity
μ	kb	Shear modulus
μ^0	kb	Shear modulus at zero-porosity
K_s	kb	Adiabatic bulk modulus
K_s^0	kb	Adiabatic bulk modulus at zero-porosity
K_T	kb	Isothermal bulk modulus
K_T^0	kb	Isothermal bulk modulus at zero-porosity
Y	kb	Young's modulus
σ_s	None	Poisson's ratio
σ_s^0	None	Poisson's ratio at zero-porosity
f_j	Hz/sec	Pulse-repetition frequency (PRF) of j th mode

f_p	Hz/sec	PRF of compressional mode
f_s	Hz/sec	PRF of shear mode
P	None	Compressional mode
S	None	Shear mode
a	None	Aspect ratio
p	kb	Pressure
T	°K	Temperature
α	per °K	Coefficient of volume expansion
θ	None	Volume fraction of pores.

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